ARX-based Cryptography

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Outline

ARX

- Addition (mod 2^n): $+, \boxplus$
- Rotation: $\lll r$
- XOR: $\oplus$

- Term ‘AXR’: Ralf-Philipp Weinmann (Dagstuhl 2009)
  - Later: renamed to ARX
- Concept of ARX is much older
  - E.g. FEAL (Eurocrypt 1987)

Advantages of ARX

- Fast performance on PCs
- Compact implementation
- Easy algorithm
- No timing attacks
- Functionally complete (assuming constant included)

Disadvantages of ARX

- Not best trade-off in hardware
- Security against linear and differential cryptanalysis?
- Security margin?
- Side-channel attacks?

ARX Designs

- Block ciphers
  - FEAL, Threefish
- Stream ciphers
  - Salsa20, ChaCha, HC-128
- Hash functions:
  - SHA-3 Finalists: BLAKE, Skein
  - SHA-3 Second Round: Blue Midnight Wish, Cubehash
  - SHA-3 First Round: EDON-$\mathcal{R}$

Designs Similar to ARX

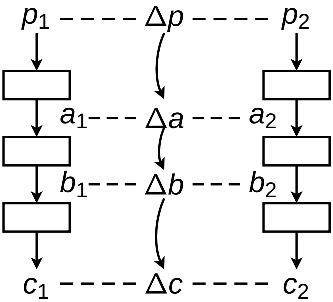
- Including left shift, right shift:
  - Block ciphers: TEA, XTEA, XXTEA
  - SHA-3 candidate: EnRUPT
- Including bitwise Boolean functions:
  - Hash functions: MD4, MD5, SHA-1
  - SHA-3 candidates: SIMD, Shabal

This presentation

- Introduce S-function concept
  - Can handle left/right shifts, bitwise Boolean functions, multiplication by constants
- Focus on differential cryptanalysis
- Analyze addition, XOR, and ARX components
- Provide observations on larger components

Differential Cryptanalysis

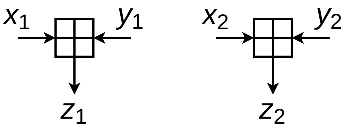
Differential characteristic: describes desired propagation of differences through cryptographic primitive



S-box vs ARX

- S-box
  - Typical size up to 8×8 bit
  - Difference distribution table: up to $2^{16} = 65536$ elements
  - Easy to calculate: differential probability, number of output differences, output difference with highest probability,...
- ARX operations
  - Typically, $n = 32$ or $n = 64$
  - Difference distribution table: 2^{64} or 2^{128} elements, too large!
  - Fast algorithms ($\mathcal{O}(n)$) required to calculate properties

xdp^+: The XOR Differential Probability of Addition



$\Delta x, \Delta y, \Delta z$ are fixed xor differences such that

$$x_2 = x_1 \oplus \Delta x, \;\; y_2 = y_1 \oplus \Delta y, \;\; z_2 = z_1 \oplus \Delta z,$$

xdp^+ expresses the fraction of pairs (x_1, y_1) for which the following holds:

$$((x_1 \oplus \Delta x) + (y_1 \oplus \Delta y)) \oplus (x_1 + y_1) = \Delta z.$$

xdp^+: Motivating Example

From “On the Additive Differential Probability of Exclusive-Or”, Lipmaa, Wallén, Dumas, FSE 2004:

$$\mathrm{xdp}^+(\mathtt{11100}, \mathtt{00110} \to \mathtt{10110})$$
$$= LA_{101}A_{100}A_{111}A_{011}A_{000}C = \frac{1}{4}$$

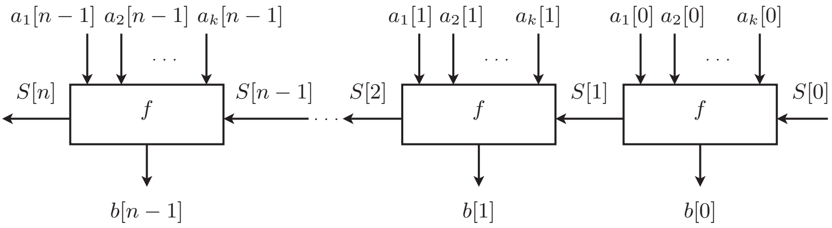
where

$$A_{000} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \quad A_{001} = A_{010} = A_{100} = \frac{1}{2}\begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix},$$
$$A_{011} = A_{101} = A_{110} = \frac{1}{2}\begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}, \quad A_{111} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix},$$
$$L = [\ 1 \quad 1\ ], \quad C = [\ 1 \quad 0\ ]^T .$$

S-function

An S-function accepts n-bit words $a_1, a_2, \ldots, a_k$ and an n-digit input state S, and produces an n-bit output word b:

$$(b[i], S[i+1]) = f(a_1[i], a_2[i], \ldots, a_k[i], S[i]), \qquad 0 \leq i < n \ .$$



xdp^+: From Words to Bits: Constructing f

$$
\begin{cases}
x_2 & \leftarrow x_1 \oplus \Delta x \\
y_2 & \leftarrow y_1 \oplus \Delta y \\
z_1 & \leftarrow x_1 + y_1 \\
z_2 & \leftarrow x_2 + y_2 \\
\Delta z & \leftarrow z_2 \oplus z_1
\end{cases}
\Longrightarrow
\begin{cases}
x_2[i] & \leftarrow x_1[i] \oplus \Delta x[i] \\
y_2[i] & \leftarrow y_1[i] \oplus \Delta y[i] \\
z_1[i] & \leftarrow x_1[i] \oplus y_1[i] \oplus c_1[i] \\
c_1[i+1] & \leftarrow (x_1[i] + y_1[i] + c_1[i]) \gg 1 \\
z_2[i] & \leftarrow x_2[i] \oplus y_2[i] \oplus c_2[i] \\
c_2[i+1] & \leftarrow (x_2[i] + y_2[i] + c_2[i]) \gg 1 \\
\Delta z[i] & \leftarrow z_2[i] \oplus z_1[i]
\end{cases}
$$

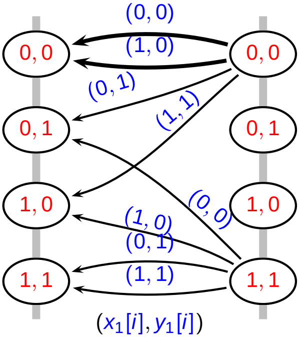
xdp^{+}: From Words to Bits: S-function

The S-function for xdp^{+} is:

$$(\Delta z[i], S[i+1]) = f(x_1[i], y_1[i], \Delta x[i], \Delta y[i], S[i]), \qquad 0 \le i < n \ ,$$
$$S[i] \leftarrow (c_1[i], c_2[i]),$$
$$S[i+1] \leftarrow (c_1[i+1], c_2[i+1]).$$

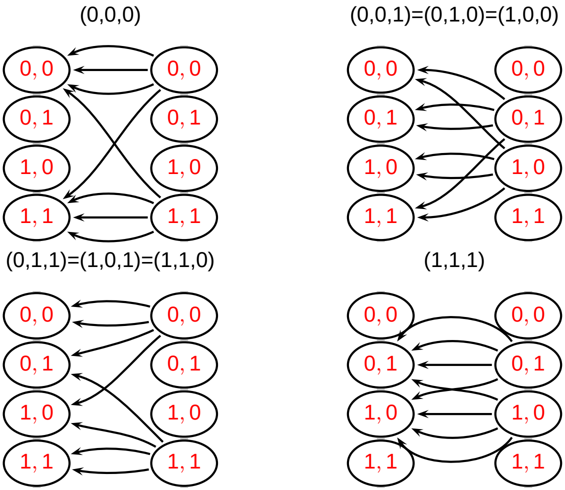
xdp$^+$: Subgraph

$(\Delta x[i], \Delta y[i], \Delta z[i]) = (1,0,1)$



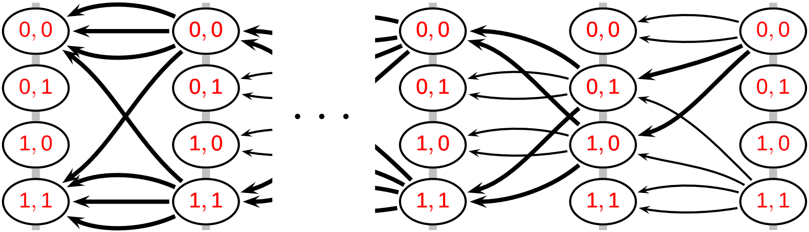
$$\begin{cases} x_2[i] & \leftarrow x_1[i] \oplus \Delta x[i] \\ y_2[i] & \leftarrow y_1[i] \oplus \Delta y[i] \\ z_1[i] & \leftarrow x_1[i] \oplus y_1[i] \oplus c_1[i] \\ c_1[i+1] & \leftarrow (x_1[i] + y_1[i] + c_1[i]) \gg 1 \\ z_2[i] & \leftarrow x_2[i] \oplus y_2[i] \oplus c_2[i] \\ c_2[i+1] & \leftarrow (x_2[i] + y_2[i] + c_2[i]) \gg 1 \\ \Delta z[i] & \leftarrow z_2[i] \oplus z_1[i] \end{cases}$$

xdp^+: All Subgraphs

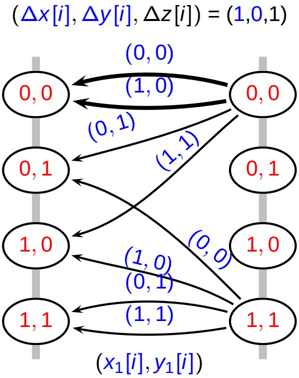


xdp^{+}: From Graphs to Probability

Computing probability xdp^{+} is equivalent to counting number of paths that satisfy $\Delta x, \Delta y, \Delta z$. Each valid pair (x_1, y_1) corresponds to path in graph (shown in bold).



xdp^+: From Subgraph to Matrix



$$S[i+1] \begin{matrix} (0,0) \\ (0,1) \\ (1,0) \\ (1,1) \end{matrix} \overset{\displaystyle S[i] \atop (0,0),(0,1),(1,0),(1,1)}{\frac{1}{4}\begin{bmatrix} \mathbf{2} & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 2 \end{bmatrix}} = A_{101}$$

xdp$^+$: All Matrices

There are four distinct matrices for xdp$^+$:
$A_{000}, A_{001} = A_{010} = A_{100}, A_{011} = A_{101} = A_{110}, A_{111}$.

$$A_{000} = \frac{1}{4}\begin{bmatrix} 3 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 3 \end{bmatrix}, \; A_{001} = \frac{1}{4}\begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix},$$

$$A_{011} = \frac{1}{4}\begin{bmatrix} 2 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 2 \end{bmatrix}, \; A_{111} = \frac{1}{4}\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 3 & 0 \\ 0 & 3 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

xdp^+: From Matrices to Probability

Computing the probability xdp^+ can be done using matrix multiplications

$$\mathrm{xdp}^+(\Delta x, \Delta y \to \Delta z) = L A_{w[n-1]} \cdots A_{w[1]} A_{w[0]} C \quad .$$

where

$$\begin{aligned} w[i] &= \Delta x[i] \parallel \Delta y[i] \parallel \Delta z[i], \;\; 0 \leq i < n, \\ L &= [\; 1 \;\; 1 \;\; \cdots \;\; 1 \;], \\ C &= [\; 1 \;\; 0 \;\; \cdots \;\; 0 \;]^T. \end{aligned}$$

xdp^{+}: Minimized Matrices

Reduce size of matrices by combining equivalent states (FSM reduction algorithm):

$$A'_{000} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \; A'_{001} = \frac{1}{2} \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix},$$
$$A'_{011} = \frac{1}{2} \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}, \; A'_{111} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}.$$

Linearization

- How to find good differential characteristics for ARX?
- Very powerful technique: linearization!
- In case of ARX: replace addition by XOR, then find low-weight codewords
- Easy to prove: $\mathrm{xdp}^+(\alpha, \beta \to \alpha \oplus \beta) > 0$

EDON-$\mathcal{R}$

- Hash function by Gligoroski et al., submission to SHA-3
- Here: analyis together with Bjørstad, unpublished

$$
\begin{array}{lllllllllllll}
T_0 & \leftarrow & (\texttt{0x55555555} & + & Y_0 & + & Y_1 & + & Y_2 & + & Y_5 & + & Y_7 \,) \ggg 0 \\
T_1 & \leftarrow & & (& Y_0 & + & Y_1 & + & Y_3 & + & Y_4 & + & Y_6 \,) \ggg 5 \\
T_2 & \leftarrow & & (& Y_0 & + & Y_1 & + & Y_2 & + & Y_3 & + & Y_5 \,) \ggg 9 \\
T_3 & \leftarrow & & (& Y_2 & + & Y_3 & + & Y_4 & + & Y_6 & + & Y_7 \,) \ggg 11 \\
T_4 & \leftarrow & & (& Y_0 & + & Y_1 & + & Y_3 & + & Y_4 & + & Y_5 \,) \ggg 15 \\
T_5 & \leftarrow & & (& Y_2 & + & Y_4 & + & Y_5 & + & Y_6 & + & Y_7 \,) \ggg 20 \\
T_6 & \leftarrow & & (& Y_1 & + & Y_2 & + & Y_5 & + & Y_6 & + & Y_7 \,) \ggg 25 \\
T_7 & \leftarrow & & (& Y_0 & + & Y_3 & + & Y_4 & + & Y_6 & + & Y_7 \,) \ggg 27
\end{array}
$$

EDON-$\mathcal{R}$

- Introduce XOR difference in bit i (i is not MSB)

$$
\begin{aligned}
T_0 &\leftarrow (\texttt{0x55555555} + Y_0 + Y_1 + Y_2 + Y_5 + Y_7) \ggg 0 \\
T_1 &\leftarrow (Y_0 + Y_1 + Y_3 + Y_4 + Y_6) \ggg 5 \\
T_2 &\leftarrow (Y_0 + Y_1 + Y_2 + Y_3 + Y_5) \ggg 9 \\
T_3 &\leftarrow (Y_2 + Y_3 + Y_4 + Y_6 + Y_7) \ggg 11 \\
T_4 &\leftarrow (Y_0 + Y_1 + Y_3 + Y_4 + Y_5) \ggg 15 \\
T_5 &\leftarrow (Y_2 + Y_4 + Y_5 + Y_6 + Y_7) \ggg 20 \\
T_6 &\leftarrow (Y_1 + Y_2 + Y_5 + Y_6 + Y_7) \ggg 25 \\
T_7 &\leftarrow (Y_0 + Y_3 + Y_4 + Y_6 + Y_7) \ggg 27
\end{aligned}
$$

EDON-$\mathcal{R}$

For a pair (a_1, a_2):

$$\Delta^{\pm}u[k] : \begin{cases} a_1[i] = 1, a_2[i] = 0, & \text{if } i = k \ , \\ a_1[i] = a_2[i], & \text{for } 0 \leq i < n, i \neq k \ . \end{cases}$$

$$\Delta^{\pm}n[k] : \begin{cases} a_1[i] = 0, a_2[i] = 1, & \text{if } i = k \ , \\ a_1[i] = a_2[i], & \text{for } 0 \leq i < n, i \neq k \ . \end{cases}$$

EDON-$\mathcal{R}$example:

$$\begin{array}{lcllllll} T_0 & = & (Y_1 & + & Y_7 & + & \ldots &) \ggg \ 0 \\ T_1 & = & (Y_1 & + & Y_4 & + & \ldots &) \ggg \ 5 \\ T_3 & = & (Y_4 & + & Y_7 & + & \ldots &) \ggg \ 11 \end{array}$$

EDON-$\mathcal{R}$

For a pair (a_1, a_2):

$$\Delta^{\pm}u[k] : \begin{cases} a_1[i] = 1, a_2[i] = 0, & \text{if } i = k \ , \\ a_1[i] = a_2[i], & \text{for } 0 \le i < n, i \ne k \ . \end{cases}$$

$$\Delta^{\pm}n[k] : \begin{cases} a_1[i] = 0, a_2[i] = 1, & \text{if } i = k \ , \\ a_1[i] = a_2[i], & \text{for } 0 \le i < n, i \ne k \ . \end{cases}$$

EDON-$\mathcal{R}$example:

$$\begin{array}{ccccccccc} 0 & = & (u[k] & + & Y_7 & + & \dots &) \ggg & 0 \\ 0 & = & (u[k] & + & Y_4 & + & \dots &) \ggg & 5 \\ 0 & = & (Y_4 & + & Y_7 & + & \dots &) \ggg & 11 \end{array}$$

EDON-$\mathcal{R}$

For a pair (a_1, a_2):

$$\Delta^{\pm}u[k] : \begin{cases} a_1[i] = 1, a_2[i] = 0, & \text{if } i = k \ , \\ a_1[i] = a_2[i], & \text{for } 0 \leq i < n, i \neq k \ . \end{cases}$$

$$\Delta^{\pm}n[k] : \begin{cases} a_1[i] = 0, a_2[i] = 1, & \text{if } i = k \ , \\ a_1[i] = a_2[i], & \text{for } 0 \leq i < n, i \neq k \ . \end{cases}$$

EDON-$\mathcal{R}$example:

$$\begin{array}{ccccccccc} 0 & = & (u[k] & + & n[k] & + & \ldots &) \ggg & 0 \\ 0 & = & (u[k] & + & Y_4 & + & \ldots &) \ggg & 5 \\ 0 & = & (Y_4 & + & n[k] & + & \ldots &) \ggg & 11 \end{array}$$

EDON-$\mathcal{R}$

For a pair (a_1, a_2):

$$\Delta^{\pm}u[k] : \begin{cases} a_1[i] = 1, a_2[i] = 0, & \text{if } i = k \ , \\ a_1[i] = a_2[i], & \text{for } 0 \leq i < n, i \neq k \ . \end{cases}$$

$$\Delta^{\pm}n[k] : \begin{cases} a_1[i] = 0, a_2[i] = 1, & \text{if } i = k \ , \\ a_1[i] = a_2[i], & \text{for } 0 \leq i < n, i \neq k \ . \end{cases}$$

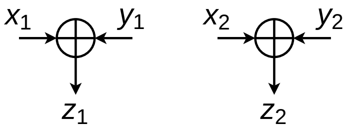
EDON-$\mathcal{R}$example:

$$\begin{array}{ccccccccc} 0 & = & (u[k] & + & n[k] & + & \ldots &) \ggg & 0 \\ 0 & = & (u[k] & + & n[k] & + & \ldots &) \ggg & 5 \\ 0 & \neq & (n[k] & + & n[k] & + & \ldots &) \ggg & 11 \end{array}$$

Linearization

- “Finding SHA-1 Characteristics: General Results and Applications”, De Cannière, Christian Rechberger, ASIACRPYT 2006
  - 64-step characteristic for SHA-1, no solution

$\mathrm{adp}^{\oplus}$: The Additive Differential Probability of XOR



$\Delta x, \Delta y, \Delta z$ are fixed additive differences such that

$$x_2 = x_1 + \Delta x, \;\; y_2 = y_1 + \Delta y, \;\; z_2 = z_1 + \Delta z,$$

$\mathrm{adp}^{\oplus}$ expresses the fraction of pairs (x_1, y_1) for which the following holds:

$$(x_1 + \Delta x) \oplus ((y_1 + \Delta y) - (x_1 \oplus y_1)) = \Delta z.$$

$\mathrm{adp}^{\oplus}$: Matrices and Probability

In a way similar to xdp^{+}, we obtain 8 matrices for $\mathrm{adp}^{\oplus}$.

$$A_{101} = \frac{1}{4} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 4 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \end{bmatrix} .$$

The probability $\mathrm{adp}^{\oplus}$ is computed again as:

$$\mathrm{adp}^{\oplus}(\Delta x, \Delta y \to \Delta z) = L A_{w[n-1]} \cdots A_{w[1]} A_{w[0]} C \quad .$$

$\mathrm{xdp}^{\times 3}$: Multiplication by 3

- Multiplication by constant: $\mathrm{xdp}^{\times C}$
  - Hash functions Shabal ($\times$3, $\times$5), EnRUPT ($\times$9)
- Let $\alpha =$ `0x12492489` and $\gamma =$ `0x3AEBAEAB`
- Approximation using xdp^{+}:

$$\mathrm{xdp}^{+}(\alpha, \alpha \lll 1 \rightarrow \gamma) = 2^{-25}$$

- Correct probability:

$$\mathrm{xdp}^{\times 3}(\alpha \rightarrow \gamma) = 2^{-15}$$

$\mathrm{xdp}^{\times 3}$: All Matrices

After minimization algorithm: 16×16 matrices reduced to 4×4:

$$A_{00} = \frac{1}{2}\begin{bmatrix} 1 & 0 & 2 & 0 \\ 0 & 0 & 0 & 2 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},\ A_{01} = \frac{1}{2}\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$A_{10} = \frac{1}{2}\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix},\ A_{11} = \frac{1}{2}\begin{bmatrix} 0 & 0 & 0 & 0 \\ 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 2 & 1 \end{bmatrix}.$$

xdc^{+}: # of Possible XOR Differentials of Addition

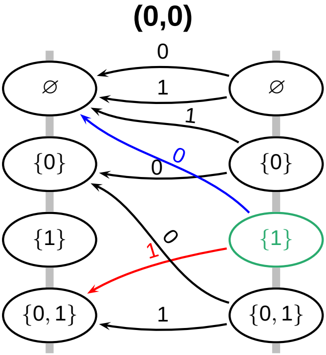
- xdc^{+} counts number of *possible* output differences, when input differences are given
- Start with minimized matrices for xdp^{+}
- Apply subset construction (automata theory)

$$\mathrm{xdc}^{+}(\Delta x, \Delta y) = L B_{w[n-1]} \cdots B_{w[1]} B_{w[0]} C \quad ,$$

where

$$w[i] = \Delta x[i] \parallel \Delta y[i], \;\; 0 \leq i < n \quad ,$$
$$L = [\begin{array}{cccc} 1 & 1 & \cdots & 1 \end{array}] \; ,$$
$$C = [\begin{array}{cccc} 1 & 0 & \cdots & 0 \end{array}]^T \quad .$$

xdc^{+}: All Possible XOR Output Differences

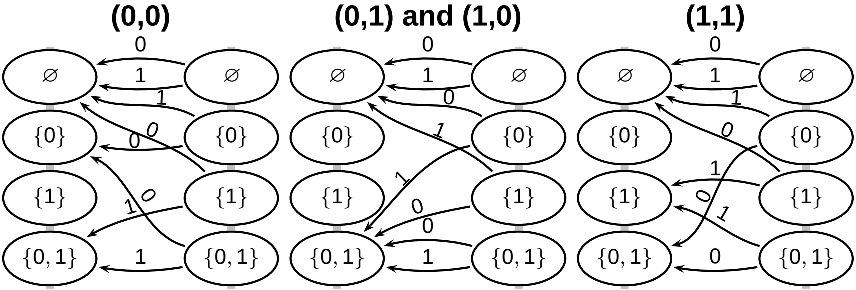


$$B_{00} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix}.$$

$$A'_{000} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix},$$

$$A'_{001} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 1 \\ 1 \end{bmatrix}.$$

xdc^{+}: Graphs



$$B_{00} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix} \quad B_{01} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 2 \end{bmatrix} \quad B_{11} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

Cryptanalysis of Hash Function Skein

- Aumasson et al. (ASIACRYPT 2009)
  - $\mathcal{O}(2^n)$ time algorithm for xdc^{+}
- Mouha et al. (SAC 2010)
  - $\mathcal{O}(n)$ time algorithm for xdc^{+}

$$\begin{aligned}
&\mathrm{xdc}^{+}(\texttt{0x1000010402000000}, \texttt{0x0000000000000000}) \\
&= L \cdot B_{00}^{3} \cdot B_{10} \cdot B_{00}^{19} \cdot B_{10} \cdot B_{00}^{5} \cdot B_{10} \cdot B_{00}^{8} \cdot B_{10} \cdot B_{00}^{25} \cdot C \\
&= 5880
\end{aligned}$$

Toolkit Available

- No need to re-implement!
- Toolkit can perform all calculations in this presentation
- Can also efficiently find maximum probability output differences (paper currently being written)

http://www.ecrypt.eu.org/tools

Ongoing Work

- Analyzing ARX as a single component – sufficient to analyze a cipher?
- Ongoing works shows not...
- Often many characteristics for same differential
- Then: Probability of differential $\neq$ Probability of characteristic

Conclusion

- ARX: Addition, Rotation, XOR
- Fast in software, increasingly used in designs
- But: security analysis seems difficult
- We need:
  - More analysis
  - Toolkits: avoid reinventing the wheel
  - Stategy for secure design